⁶Bradshaw, P., "Complex Turbulent Flows," *Journal of Fluids Engineering*, Vol. 97, June 1975, pp. 146–154.

⁷Birch, S. F., "Turbulence Models—Progress and Problems," *Computer Methods in Fluids*, edited by K. Morgan, C. Taylor, and C. A. Brebbia, Pentech, London, 1980, pp. 285–307.

⁸Rubel, A., "On the Vortex Stretching Modification of the $k-\varepsilon$ Model: Radial Jets," *AIAA Journal*, Vol. 23, No. 7, 1985, pp.1129, 1130.

⁹Sarkar, S., "The Stabilizing Influence of Compressibility on Turbulence in High-Speed Shear Flows," AIAA Paper 94-2243, June 1994.

¹⁰ Sarkar, S., "The Stabilizing Effect of Compressibility in Turbulent Shear Flows," *Journal of Fluid Mechanics*, Vol. 282, Jan. 1995, pp. 163–186.

¹¹ Vreman, A. W., Sandham, N. D., and Luo, K. H., "Compressible Mixing Layer Growth Rate and Turbulence Characteristics," *Journal of Fluid Mechanics*, Vol. 320, Aug. 1996, pp.235–258.

¹² Sfeir, A. A., "The Velocity and Temperature Fields of Rectangular Jets," *International Journal of Heat and Mass Transfer*, Vol. 19, No. 11, 1976, pp. 1289–1297

¹³Gutmark, E., Bowman, H. L., and Schadow, K. C., "Effect of Nozzle Design on Near Acoustic Field of Supersonic Circular and Rectangular Jets," AIAA 14th Aeroacoustics Conf., 1992.

¹⁴McGuirk, J. J., and Rodi, W., "The Calculation of Three-Dimensional Turbulent Jets," *Proceedings of the Symposium on Turbulent Shear Flows*, Vol. 1, 1977, pp. 1.29–1.36.

¹⁵Tjonneland, E., and Birch, S. F., "Application of Viscous Analyses to the Design of Jet Exhaust Powered Lift Installations," *Journal of Engineering for Power*, Vol. 102, No. 3, 1980, pp. 626–631.

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Reply by the Authors to S. F. Birch

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In N his comment on our paper, Birch states that he does not think that the Pope² and Sarkar³ modifications represent the correct physics of turbulent mixing in jets. He indicates that the modifications should not be used. However, he offers no alternative, even though it is well documented that the standard $K-\varepsilon$ model does

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not predict three-dimensional high-speed jet flows well. Further, he notes that for jets issuing from nozzles with complex geometries or complicated upstream flow conditions, accurate initial conditions for parabolized computations are important.

When Pope² formulated his correction term, he did appeal to the notion of vortex stretching. However, his correction may be viewed merely as a way to account for the reduction in turbulence levels in three-dimensional jet flows compared to two-dimensional shear layers. This view is the one expressed in our paper. We attribute the reduced turbulence levels to the behavior of the large-scale structures and not to some general result of flow divergence. The effect of the correction is that the spreading rates of axisymmetric and three-dimensional jets are smaller than that of the two-dimensional mixing layer. This matter was discussed in our paper. The jet flow results we obtained by using the correction are excellent when compared with experiments. We have made no recommendation regarding the use of Pope's correction in other environments (e.g., wall-bounded flows and radial jets).

In our paper, we discussed the physical mechanism⁴ that causes the reduction in high-speed mixing. This mechanism is not included in the standard $K-\varepsilon$ model. It is obvious that a modification to the $K-\varepsilon$ model is needed if this effect is to be reproduced in the calculations. In any model, including every turbulence model, the objective is to have a simple mathematical representation that does what the physical mechanism does. We did note the fact that the Sarkar model was derived on a different physical basis. However, upon close examination, we find that the model does simulate the characteristics of the actual physical mechanism in jets. We elaborated on this issue in the paper.

A parabolized computation can be accurately applied to very complicated flows. However, it should be applied only to flows with very little upstream influence. For most jet flows, this condition is well satisfied some distance downstream of the nozzle exit. In our calculations, initial conditions are chosen to match the data at some downstream location. In each case, the data vary slowly (i.e., exhibit very little upstream influence) downstream of the first measurement point. As a result, the parabolized calculations are quite appropriate for the jet flows considered. We certainly agree that it would be foolish to use parabolized calculations for flows with significant upstream influence or for which appropriate initial conditions cannot be estimated.

References

¹Thies, A. T., and Tam, C. K. W., "Computation of Turbulent Axisymmetric and Nonaxisymmetric Jet Flows Using the K− ε Model," *AIAA Journal*, Vol. 34, No. 2, 1996, pp. 309–316.

²Pope, S. B., "An Explanation of the Turbulent Round-Jet/Plane-Jet Anomaly," *AIAA Journal*, Vol. 16, No. 3, 1978, pp. 279–281.

³Sarkar, S., and Lakshmanon, B., "Application of a Reynolds Stress Turbulence Model to the Compressible Shear Layer," *AIAA Journal*, Vol. 29, No. 5, 1991, pp. 743–749.

⁴Papamoschou, D., and Roshko, A., "The Compressible Turbulent Shear Layer," *Fluid Mechanics*, Vol. 197, Dec. 1988, pp. 453–477.

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